University of Paris 1 Year 2014 - 2015

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# Evaluation 3

Let $\mathcal{R}$ be the relation on $\mathbb{Z}$ defined by

$$x\mathcal{R}y \quad \Longleftrightarrow \quad \exists k \in \mathbb{Z},\, x - y = 2k.$$

1. Prove that $\mathcal{R}$ is an equivalence relation. (3pt)
2. What are the elements of $\mathbb{Z}/\mathcal{R}$? (3pt)
3. Prove that $\mathcal{R}(0) \cap \mathcal{R}(1) = \emptyset$? (2pt)
4. What is $\inf\{\mathcal{R}(0), \mathcal{R}(1)\}$ and $\sup\{\mathcal{R}(0), \mathcal{R}(1)\}$ for the order relation $\subseteq$ on $2^{\mathbb{Z}}$? (3pt)
5. Let $f : \mathbb{Z} \to \mathbb{Z}/\mathcal{R}$.
   (a) Prove that $f$ is never bijective. (2pt).
   (b) Prove that if $f$ is such that $\exists x, y \in \mathbb{Z}, f(x) \neq f(y)$ then $f$ is surjective. (2pt)
   (c) Find a function $f : \mathbb{Z} \to \mathbb{Z}/\mathcal{R}$ which is neither injective nor surjective. (2pt)